

# Testing Gravity with the CFHTLS-Wide Cosmic Shear Survey and SDSS LRGs

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General relativity as one the pillar of modern cosmology has to be thoroughly tested if we want to achieve an accurate cosmology. We present the results from such a test on cosmological scales using cosmic shear and galaxy clustering measurements. We parametrize potential deviation from general relativity as a modification to the cosmological Poisson equation. We consider two models relevant either for some linearized theory of massive gravity or for the physics of extra-dimensions. We use the latest observations from the CFHTLS-Wide survey and the SDSS survey to set our constraints. We do not find any deviation from general relativity on scales between 0.04 and 10 Mpc. We derive constraints on the graviton mass in a restricted class of model.

## I. INTRODUCTION

Observational cosmology has established the flat  $\Lambda$ CDM model as the standard model of modern cosmology [1, 2, 3, 4, 5, 6, 7, 8]. Within this model the energy budget of the Universe roughly goes as follows : 4% comes from baryons, 20% from Cold Dark Matter (CDM) and 76% is in the form of a cosmological constant. Those numbers are now known to a few percent but despite those tight constraints, the exact nature of CDM is still unknown and the high value of the cosmological constant –or more generally Dark Energy (DE)– keeps challenging our deepest understanding of fundamental physics [9]. The unknown nature of the required last two components might cast some doubts on the foundation of this model : the linear cosmological perturbation theory in General Relativity (GR). Although GR passes direct tests probing the Solar System scales ( $10^{11}$ m) down to the laboratory scales ( $10^{-3}$ m) [10], extrapolating its validity on more than 10 orders of magnitude to address cosmological scales ( $10^{26}$ m) is questionable. GR thus has to be checked on all relevant cosmological scales in as many ways as possible [11]. We present in this paper such a test on megaparsec scales using two probes of the large scale structures of the Universe : galaxy clustering and weak gravitational lensing.

Our approach is phenomenological and we do not aim at developing or at testing specific fully consistent alternatives to GR. Several classes of theory have been studied in details in the literature with the aim at accounting for either the acceleration of the Universe or for Dark Matter or both. The first class includes for example, a five-dimensional alternative [12, 13] (DGP) or the addition of non-linear terms in the Ricci scalar to the gravitational action [14, 15, 16, 17, 18]. The second class (for now) includes tensor-vector-scalar the-

ory [19, 20, 21, 22, 23, 24, 25, 26] as a covariant theory encompassing the MOND theories [27]. Finally, the last class includes for example the ghost condensate theory [28]. Generic models that verify Birkhoff's theorem were also investigated in [29]. Instead of exploring these new theories, we rather propose simple functional forms parametrizing the deviation from gravity on Mpc scales and constrain them using current observational data at low redshift. If any significant deviation from gravity was observed, we would then make the connection with theoretical models more explicit. Our key assumption is that the deviation from GR we explore can be written as a modification to the Poisson equation. As such we are testing the Poisson equation on cosmological scales.

Several authors already tackled the tasks of constraining alternate theories of gravity with available or planned observations [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51]. In particular, within the context of linearized relativity, Edery [52] showed that it is not possible to build a relativistic theory that reproduces galactic phenomenology (flat rotation curves, gravitational lensing, or dynamical mass measurements via the virial theorem) without introducing dark matter (their key point is that the light deflection cannot be made to have the right sign). Following a similar approach, Zhytnikov and Nester [53] consider a general metric theory that would reproduce the dynamics of galactic systems without dark matter and reach the same conclusion. They advocate in particular the addition of a Yukawa like potential to the usual Newtonian one as a generic extension for a metric theory of gravity. White and Kochanek [54] then considered the same extension to GR gravity, but focussed on scales relevant to weak-lensing (note that they only discussed the effect on the deflection angle and not on the growth rate of structure as we will do below). As they discuss, a

generic feature of such linearized extension to GR is to modify the standard Poisson equation relating the density field to the gravitational potential. In turn, Uzan and Bernardeau [55] considered an alternative modification to the Poisson equation that encompasses DGP theories. Shirata et al. [56] and Sealfon et al. [57] constrained the Yukawa type extension to GR using galaxy clustering measurements. Note that Sealfon et al. [57] also introduced a power-law extension to gravity. Such modifications to the Poisson equation were later studied using numerical simulations [45, 47].

Our work is an extension to the latter works [45, 47, 54, 55, 56, 57]. In particular, we use the latest weak gravitational lensing and clustering data to test gravity on Mpc scales at low redshift. The two alternate theories of gravity we constrain are introduced in Sec. II, before detailing their phenomenology. We then describe our data-sets and methodology in Sec. III before presenting and discussing our results in Sec. IV.

## II. MODIFIED THEORIES OF GRAVITY AND CONSTRAINT METHODS

### A. Theoretical motivations

To allow for deviations from general relativity opens up many possibilities and a wide range of gravity theories that we cannot explore exhaustively. We restrict ourselves to a few somewhat phenomenological models that offer both a physical motivation and observationally tractable cosmological implications.

We will consider two different models that we introduce below: the Yukawa and the Uzan-Bernardeau type models.

As White and Kochanek [54], we follow Zhytnikov and Nester [53] who within well defined postulates presents some general arguments regarding the shape of a linearized metric theory of gravitation. These postulates comprise post-Newtonian slow motion extension ( $(v/c)^2 \ll 1$ ) and weak gravitational field regime relevant to the scale we consider in this work. Their description includes forces mediated by massless or massive scalar and tensor modes. The metric reduces to

$$g_{00} = (-1 + 2\Phi) \quad (1)$$

$$g_{ij} = (+1 + 2\Phi)\delta_{ij} \quad (2)$$

where the potential  $\Phi$  is given by

$$\Phi(\mathbf{r}) = (1 - \alpha)\Phi(\mathbf{r}, 0) + \alpha\Phi(\mathbf{r}, m) \quad (3)$$

$$\Phi(\mathbf{r}, m) = G \int \frac{\rho(\mathbf{r}') d^3\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} e^{-m|\mathbf{r} - \mathbf{r}'|}. \quad (4)$$

Thus, to the usual Newtonian potential  $\Phi(\mathbf{r}, 0)$  a Yukawa type potential  $\Phi(\mathbf{r}, m)$  is added corresponding to propagating massive modes.

Considering the evolution of over-density,  $\delta(\mathbf{r}, t)$  or its Fourier transform,  $\tilde{\delta}(\mathbf{k}, t)$  the standard linear theory of

perturbations leads to the Poisson equation relating the gravitational potential to the over-density. The Poisson equation writes in comoving coordinates and Fourier space [58]

$$\tilde{\Phi}(\mathbf{k}, a) = -\frac{3}{2} \frac{H_0^2 \Omega_{m0}}{a} \frac{\tilde{\delta}(\mathbf{k}, t)}{k^2} f(\mathbf{k}, a) \quad (5)$$

where  $a$  is the scale factor,  $H_0$  is the Hubble constant,  $\Omega_{m0}$  the present matter density.  $f$  is a deterministic function equal to 1 for the standard gravity. It can easily be shown that the inclusion of a Yukawa type potential as in Eq. 4 leads to

$$f^{Yuk}(\mathbf{k}) \equiv f(\mathbf{k}) = 1 - \alpha \frac{1}{1 + \left(\frac{k}{a m}\right)^2}. \quad (6)$$

As such, exploring alternative theories of gravity will be equivalent for us to test the Poisson equation.

Observational constraints on similar models (but with slightly different notations) coming from galaxy surveys [56, 57], as well as using also their non-linear evolution using N-body simulations [45, 47], have already been studied. Note that our notation matches that of White and Kochanek [54] and corresponds to that of Shirata et al. [56] and Sealfon et al. [57] with ( $\alpha \rightarrow -\alpha$ ,  $\lambda \rightarrow -1/m$ ) and [45] with ( $\alpha \rightarrow -\alpha$ ,  $\lambda \rightarrow -1/r_s$ ).

Another set of models captures, in the context of superstring theories, some brane induced phenomenology. In such a scenario, a generic feature seems to be the existence of two scales below/above which standard gravity is altered [59, 60, 61, 62]. The smaller scale (of order a millimeter or less), irrelevant to our measurements, corresponds to the existence of Kaluza-Klein gravitons. On the other hand, above the branes separation scale, we also expect gravity to be altered. Since this scale is exponentially larger than the previous one, it becomes cosmological. The use of large scale structures as a probe of this form of deviation from standard gravity as been advocated by [29, 55, 63], who include the model of [62]. Uzan and Bernardeau [55] focus in particular on weak gravitational lensing, as such we will follow their work more closely. They describe in real space the violation of Newton's law above a given physical scale,  $r_s$ , as a multiplicative function to the standard Newtonian potential,  $\Phi(r, 0)$ ,

$$\Phi(r) = \Phi(r, 0) \frac{1}{1 + \frac{r}{r_s}} \quad (7)$$

so that on small scales,  $r \ll r_s$ , we recover the usual Newtonian gravity. As before, this translates into a modification to the Poisson equation with a multiplicative function,  $f^{UB}$  [47, 55]

$$f^{UB}(k, a, r_s) \equiv f(k, a) \quad (8)$$

$$= \frac{kr_s}{2a} \left[ -2 \sin(kr_s/a) \int_{kr_s/a}^{\infty} \frac{\cos(t)}{t} dt + \cos(kr_s/a) \left( \pi - 2 \int_0^{kr_s/a} \frac{\sin(t)}{t} dt \right) \right]. \quad (9)$$

$r_s$  is an arbitrary scale and is the parameter we will constrain later on.

From now on, we will focus on these two models (Yukawa and UB) and study their phenomenological implications on cosmological scales ( $\simeq 10$  Mpc) before constraining their free parameters. We deliberately restrict our conclusions from those models to the scales we are probing, *i.e.* we do not explore the parameter space where  $1/m$  or  $r_s$  are close to the Horizon scale. Note that we choose  $1/m$  and  $r_s$  to be physical distances, *i.e.* we do not allow them to be comoving distances.

## B. Phenomenology

Modifications to the Poisson equation as described above translate into an alteration of gravity above a characteristic scale ( $1/m$  or  $r_s$ ). To illustrate this we choose to focus on the linear perturbation theory and compute the linear growth rate of structure. We do not attempt at building a fully covariant theory of gravity (a task well beyond the scope of this paper) and therefore we consider that the evolution of the background metric will be the one of the  $\Lambda$ CDM model currently favored by the data (See however [64, 65] for a discussion of background evolutions). Our motivation to do so relies on the fact that any alternative theory of gravity will have to reproduce the now well observationally established  $\Lambda$ CDM background evolution. It seems thus fair to keep using it as an effective description of the evolution of the background metric. However, if our test of gravity on 10 Mpc scales or so were to detect any deviation, a more acute theoretical description would be required.

A modification to the Poisson equation leads to a change on the linear growth rate of perturbations. If  $D$  is defined so that  $\delta(\mathbf{k}, t) = D(\mathbf{k}, t)\delta(\mathbf{k}, t_0)$  where  $t_0$  is an arbitrary original time [56, 58], then it satisfies the equation

$$\ddot{D} + 2H\dot{D} = \frac{3}{2} \frac{H_0^2 \Omega_{m0}}{a^3} f(k) D \quad (10)$$

where “ $\dot{\phantom{x}}$ ” denotes time derivative and where we restrict ourselves to time independent  $f$  and  $k$  is the comoving wave number. The Hubble parameter,  $H$ , and its evolution are defined as for the  $\Lambda$ CDM standard case by

$$H = \frac{\dot{a}}{a} \quad (11)$$

$$H^2(a) = H_0^2 \left( \frac{\Omega_{m0}}{a^3} + \Omega_\Lambda \right). \quad (12)$$

Solving Eq. 10 numerically is straightforward. To fix boundary conditions, by analogy with the standard case, we consider the limit  $a \rightarrow 0$  and look for a solution  $D = a^n$ . Using the fact that  $H^2 \rightarrow H_0^2 \Omega_0/a^3$  (we tend to Einstein-de Sitter) when  $a \rightarrow 0$  we find that the growing mode solution is given by

$$n = \frac{1}{4} \left( \sqrt{1 + 24f(k)} - 1 \right) \text{ if } 1 + 24f(k) \geq 0. \quad (13)$$

Numerical solutions to this equation at  $z = 0$  for various parameters are plotted in Fig. 1, or more precisely the growth rate for modified gravity divided by the expected growth rate of structures for a  $\Lambda$ CDM model. Let’s discuss the Yukawa model first (left panel). As visible in Eq. 4, the Yukawa potential introduces a damping on gravity for  $r \gg 1/m$  which entails a slow down of structure growth. More generally, an increase or a decrease of gravity is expected, depending on the sign of  $\alpha$ . This is visible in the left panel of Fig. 1 for various  $\alpha$ s and  $ms$ . By construction, our models converge toward standard gravity at the smallest scales ( $f \rightarrow 1$  when  $k \rightarrow \infty$ ) and tends to a weaker but scale independent gravity for  $\frac{k}{ma} \ll 1$  hence the flat limit for low  $k$ . The amplitude of this plateau depends only on  $\alpha$ . The on-set of the transition is however controlled by both  $\alpha$  and  $m$  and, for a given  $\alpha$ , the smaller  $m$  the lower in  $k$  the transition occurs. For a given  $m$  however, the smaller  $|\alpha|$ , the lower in  $k$  is the departure from standard gravity. Those results agree with previous published results, as *e.g.* Stabenau and Jain [45] and the analytical estimates of Sereno and Peacock [44].

The second model we consider is the UB model, as defined in Eq. 9. It only depends on the cut-off scale  $r_s$  and as expected, the growth of structures for scales larger than  $r_s$  will be slower as compared to  $\Lambda$ CDM as the gravity gets weaker. This is clearly visible in the second panel of Fig. 1.

## III. ANALYSIS METHODOLOGY

### A. Observables

Making use of the previously derived linear theory predictions, we now constrain our models using two probes of the growth rate of large scale structures: cosmic shear and galaxy clustering.

For the former we will use the latest Canada-France-Hawaii-Telescope Legacy Survey (CFHTLS [82]) observations as in Fu et al. [66], a work extending the analyses of the previous release [67, 68, 69]. For the latter we will use the Sloan Digital Sky Survey (SDSS [83]) matter power spectrum estimated from the clustering of Luminous Red Galaxies as in Tegmark *et al.* [70].

Since weak gravitational lensing provides a mean to directly image the total mass distribution as a function of redshift, it is potentially a powerful way to constrain accurately the growth of structures and thus any theory affecting it. The data we use here are based on the recently published analysis of the CFHTLS-Wide survey (release T0003) that spreads over 57 square degrees (34.2 after masking). The depth of the weak lensing catalog reaches a magnitude of  $i'_{AB} = 24.5$  and corresponds to a galaxy density of about 13.3 gal./arcmin<sup>2</sup>. It comprises an effective sample of about  $1.7 \times 10^6$  galaxies whose shape correlation properties have been analysed by Fu et al. [66]. Although five bands will eventually be available, only the

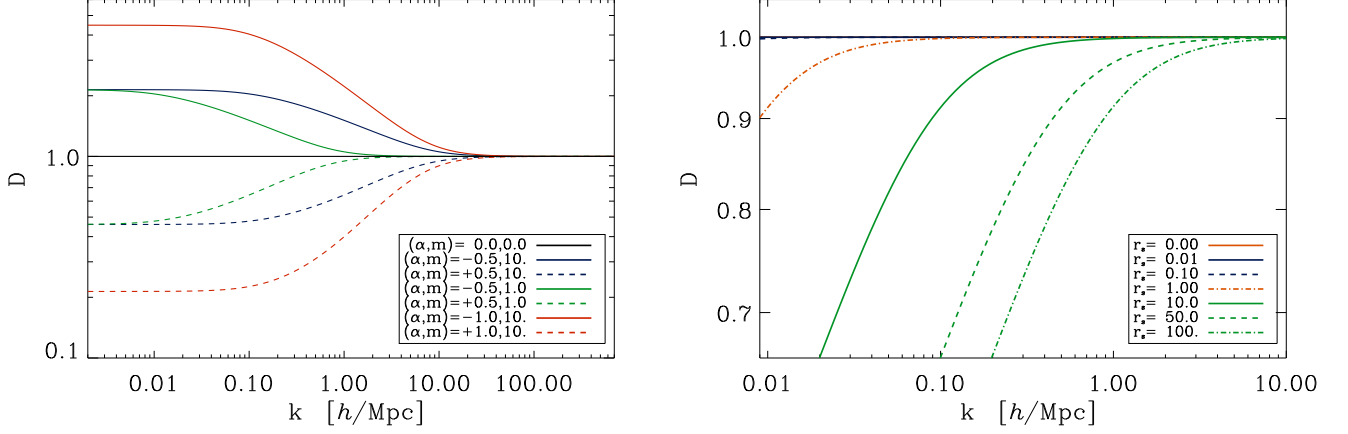


FIG. 1: Ratio of linear growth factor of modified gravity versus  $\Lambda$ CDM at  $z = 0$ . From left to right, the Yukawa model and the UB model.

$i'$  band was used for this analysis. The area used to produce this data-set is about twice larger than the previous release of the CFHTLS (but only 35% of the total size of the survey) but explores much larger scales. The width of this area turns out to be crucial to our analysis since the cosmic shear is now measured from 1 arcmin up to 4 degree, well into the linear regime. Since we are looking for variation in the shape of the power spectrum, this wide range of scales is particularly valuable. Although it would be highly beneficial to our project, no tomography measurements has been carried out with these data yet. Systematic effects are constrained to be smaller than statistical errors. “B-modes” in particular are negligible at all scales we use. To constrain gravity, we choose two lensing statistics whose properties are slightly different : the shear E/B correlation functions,  $\xi_E$ , and the compensated filter known as aperture mass statistic,  $M_{ap}$  [66, 71]. The former is defined as

$$\xi_E(\theta) = \frac{1}{2\pi} \int_0^\infty dk k P_\kappa(k) J_0(k\theta) \quad (14)$$

whereas the latter is defined as

$$M_{ap}^2(\theta) = \frac{288}{\pi\theta^4} \int_0^\infty \frac{dk}{k^3} P_\kappa(k) J_4^2(k\theta) \quad (15)$$

where

$$P_\kappa(\ell) = \frac{9H_0^4\Omega^2}{4c^4} \int_0^{\omega_H} d\omega \left( \frac{W(\omega)}{a(\omega)} \right)^2 D(k, \omega) P_\delta(\ell/f_K(\ell)). \quad (16)$$

$P_\delta$  is the density power spectrum at  $z = 0$  and the geometry factor  $W$  is defined as

$$W(\omega) = \int_w^{\omega_H} dw' n(w') \frac{f_K(w' - w)}{f_K(w')} \quad (17)$$

with the redshift distribution given by

$$n(z) = \frac{\beta}{z_s \Gamma\left(\frac{1+\alpha}{\beta}\right)} \left(\frac{z}{z_s}\right)^\alpha \exp\left[-\left(\frac{z}{z_s}\right)^\beta\right] \quad (18)$$

and where  $f_K$  is the comoving angular diameter distance. Both statistics involve a different weighting of the convergence power spectrum depending on whether a wide or narrow kernel is favored in real or Fourier space [72]. As illustrated in [66], the use of two different statistics provide an extra consistency check for the measurements and their interpretation.

We assumed here that the estimators have been properly calibrated and that  $\xi_+ = \xi_E$  ( in the notations of [66, 69]). As Fu et al. [66] we will discard the four smallest angular scales when using the  $M_{ap}^2$  statistic due to excessive E/B mixing. We will use the exponential redshift distribution defined in Eq. 18 as Benjamin et al. [69]. It contains three free parameters  $z_s$ ,  $\alpha$  and  $\beta$  that are determined using the photometric redshift calibrated on the VIRMOS VLT Deep Survey [73]. The uncertainties in  $\alpha$  and  $\beta$  are sufficiently small that we can fix  $\alpha$  and  $\beta$  to respectively 0.838 and 3.43. However we still need to marginalize over the  $z_s$  uncertainties, as it will be described in the next sub-section. The choice of the Benjamin et al. [69] redshift distribution instead of the Fu et al. [66] seems somewhat inconsistent and may look like a useless complication. It is primarily motivated by the huge computational gain in keeping  $\alpha$  and  $\beta$  constant. Although the Benjamin et al. [69] distribution is slightly less accurate than the power-law one used in Fu et al. [66], the few percent difference is not relevant for our purpose but it would be if we were trying *e.g.* to constrain the overall amplitude instead of marginalizing over it.

The large-scale real-space power spectrum  $P_\delta(k)$  is measured using a sample of  $2 \times 10^6$  galaxies from the

Sloan Digital Sky Survey, covering about  $2 \times 10^3$  square degrees with median redshift  $z \simeq 0.35$ . We follow the methodology described in Tegmark *et al.* [70] and use the likelihood code available at [84]. Note that these measurements of the real-space matter power spectrum  $P(k)$  are up to an unknown overall multiplicative bias factor,  $b$ , over which we will marginalize as described below. This bias is assumed to be linear, *i.e.* constant, in the range probed by the SDSS and CFHTLS-Wide data sets. A strong scale dependence, although unlikely on those scales, could potentially be degenerate with the effects we are looking at.

One subtlety arises when computing the density power spectrum required to derive theoretical expectations for these observables. In both cases, to define an initial power spectrum we use the fitting formula of Novosyadlyj *et al.* [74] as used by Tegmark *et al.* [70]. We normalize it using  $A_s$ , the amplitude of density fluctuations at  $k = 0.05 \text{ Mpc}^{-1}$  (see [75]). We then modify the growth rate as a function of  $k$  and  $a$  according to the numerical results of Eq. 10. Since the galaxy clustering data we are using are in the mildly non-linear regime ( $k \leq 0.2h/\text{Mpc}$ ), we do not apply any non-linearity corrections besides the redshift space distortion one as in Tegmark *et al.* [70]. The situation is different for cosmic shear. Schematically, the smallest angular scale of interest to our lensing data is 1. arcmin. Neglecting projection effects, this angular scale corresponds to about  $0.6 \text{ Mpc}/h$  at the median redshift of our lens population ( $z = 0.5$ ), that is around  $k \simeq 2h/\text{Mpc}$ . At these scales, the non-linear corrections to the power spectrum are of order a few and thus non-negligible. However, in the standard  $\Lambda\text{CDM}$  model, these corrections are computed using fitting formulae calibrated on numerical simulations [76, 77] that use standard gravity. Their relevance to the alternative theory of gravity we consider is therefore not obvious. This loop hole has already been tackled in the literature and does not turn out to be a critical issue. Stabenau and Jain [45] used numerical simulations with modified gravity and found that in the context of our Yukawa model, both prescriptions give reasonable fits. Peacock and Dodds [76] seems to provide a better fit for negative  $\alpha$  (our conventions), whereas for positive  $\alpha$ , both prescriptions are as good. These results have recently been confirmed and extended to include the Uzan and Bernardeau [55] model we consider [47]. As a precautionary measure, all our results will use both prescriptions.

## B. Likelihood methodology and priors

We set up to constrain these alternative theories of gravity using maximum likelihood methods. We choose to maximize the likelihood in a five (four) dimensional parameter space. Two (one) of these parameters are the prime physical parameters we are interested in, be it either  $(\alpha, m)$  in the Yukawa case ( $(r_s)$  in the UB

case). We then choose an amplitude parameter  $A_s$  corresponding to the normalization of density perturbations at  $k=0.05\text{Mpc}^{-1}$ . This parameter is tightly constrained by WMAP 3 year measurements only, but using WMAP as a prior seems inappropriate in our case. The CMB constraints come from much larger scales than the upper limits of CFHTLS-Wide and SDSS data, typically a few hundreds of  $\text{Mpc}/h$ . Since our modified gravity models affect gravity and so the matter power spectrum on scales larger than a characteristic scale – be it  $1/m$  or  $r_s$  – that far exceed the 0.1-10  $\text{Mpc}$  range of the data, it is not suitable to apply a CMB prior on  $A_s$ . Instead, it is preferable to normalize the power spectrum on smaller scales. Such a normalization is available through the joint measurement of the  $Ly_\alpha$  forest flux fluctuations with other constraints on the linear matter density power spectrum *e.g.* SDSS [78]. Seljak *et al.* [79] showed that at the  $2\sigma$  level, the WMAP 3 years and small scale  $Ly_\alpha$  constraints are consistent. As such, we will use conservatively a prior on  $A_s$  derived from the WMAP and  $Ly_\alpha$  joint measurements : we assume a uniform prior for  $A_s$  in between the 99.7%CL of WMAP 3 year only [1] :  $2.988 \leq \log(10^{10} A_s (k = 0.002 \text{ Mpc}^{-1})) \leq 3.324$ . Note that because of the altered growth of structures, the scaling  $\sigma_8$  with  $A_s$  will depend on  $(\alpha, m)$  or  $(r_s)$ . This choice of normalization does not matter for the SDSS data, because of the unknown bias, but is more critical for weak-lensing observations.

Finally, we have to take into account two extra nuisance parameters. For weak lensing observables, we marginalize over the parameter  $z_s$  of the source distribution (Eq. 18) assuming a uniform prior within the  $2\sigma$  constraints of Benjamin *et al.* [69] :  $z_s = 1.172 \pm 0.026$ . We also marginalize the SDSS likelihood over the bias parameter assuming a uniform prior  $0.25 < b < 4$  which comprises most reasonable solutions [56].

The limited range in scales of our observations leads us to restrict ourselves to the following range for our prime parameters. We explore  $-0.5 < \alpha < 0.7$  and  $0.001 < m < 30$  in the case of Yukawa, and  $0.001 < r_s < 10$  in the case of UB.

Finally, we fix the other cosmological parameters to their best fit flat  $\Lambda\text{CDM}$  WMAP value only (Spergel *et al.* [1]). Namely we consider  $h = 0.732$ ,  $\omega_m = 0.1277$ ,  $\omega_b = 0.0223$  and  $n_s = 0.958$ . We did not explore the dependence of our results with regards to those parameters although we expect it to be small given the small uncertainties with which those parameters are now measured [57]. Those parameters essentially fix the background evolution and the initial power spectrum.

We explore the likelihood in this five dimensional space using a regular linear gridding of this hyper-volume (except for the  $m$  dimension that we explore using a logarithmic binning), with typically 100 samples per direction. Our code makes use of the existence of fast dimensions  $b$  that require very little computation to be explored so that we can explore this full hyper-volume in 10 hours or so using 8 1.6GHz Intel Xeon cpus.

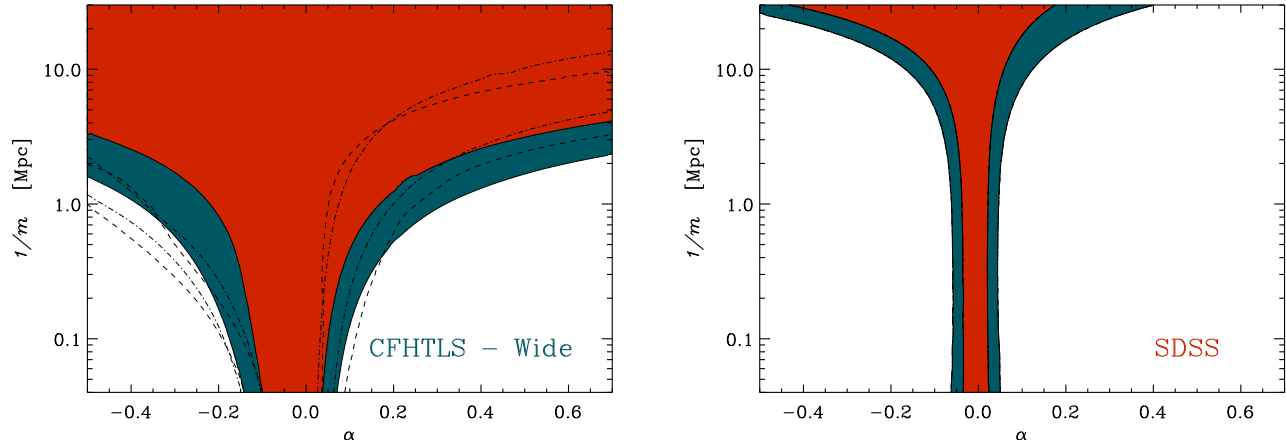


FIG. 2: Likelihood contours at 68% and 95% confidence levels for the  $1/m$  and  $\alpha$  parameters of the Yukawa type modification to gravity. The left panel corresponds to the CFHTLS-Wide constraints while the right panel corresponds to SDSS LRGs. Colored contours correspond for CFHTLS-Wide to the use of the  $M_{ap}^2$  statistic with the `halofit` non-linear prescription. The dashed lines were obtained using the  $M_{ap}^2$  statistic with the Peacock and Dodds [76] prescription whereas the dot-dashed lines were obtained with the `halofit` non-linear prescription but using the  $\xi_E$  statistic. The agreement between these various prescriptions and statistics is a satisfying of robustness of our measurement. As expected given the wider area covered by SDSS (42 times bigger than the current status of CFHTLS-Wide), the SDSS constraints are much narrower despite the bias uncertainty.

#### IV. RESULTS AND DISCUSSIONS

Fig. 2 displays the 2D 68% and 95% contour levels for the Yukawa model  $m^{-1}$  and  $\alpha$  parameters using CFHTLS-Wide (left panel) and SDSS LRGs (right panel). Other parameters have been marginalized over. Focusing on the CFHTLS panel first, we display several set of contours. The colored contours correspond to the  $M_{ap}^2$  statistic (Eq. 15) with the Smith et al. [77] `halofit` prescription to model non-linearities. The long dashed lines correspond to the  $M_{ap}^2$  statistic with the Peacock and Dodds [76] prescription. The dot-dashed lines correspond to the  $\xi_E$  statistic (Eq. 14) with the Smith et al. [77] `halofit` prescription. To the level of accuracy we are interested in this work, this figure shows that those various methods and measurements are consistent with one another. This is a reassuring statement regarding the robustness of our constraints. We will from now on quote numbers from the  $M_{ap}^2$  statistic using the `halofit` prescription (colored contours).

In the case of Yukawa type models, for a given  $1/m$  the deviation from gravity increases with increasing  $|\alpha|$  (see Fig. 1) hence it is expected constraints be centered on our favored value  $\alpha = 0$ . Therefore, Fig. 2 first shows that no deviation from standard gravity is favored by CFHTLS data. Since weak-lensing constrains small scales best (small  $1/m$ ), we observe narrower constraints at lower  $1/m$ . The broadening of the contours at high  $1/m$  shows the limits of our data in terms of large scale sensitivity. Below this broadening, say below  $1/m \leq 0.5$  Mpc, we can see that  $\alpha$  and  $1/m$  are decorrelated, *i.e.* the constraints on  $\alpha$  seem independent of  $1/m$  and vice versa. This can

be understood as follows. For a given  $1/m$ , the transition to modified gravity is set and  $\alpha$  will modify the amplitude of the effect on the linear growth rate as visible in the right panel of Fig. 1. Since for weak-lensing the relevant quantity is a weighted projection of the linearly evolved power spectrum (with non-linear corrections applied for each  $z$  considered), we expect a degeneracy between  $\alpha$  and the overall amplitude of the power spectrum set by  $A_s$ . This is illustrated in Fig. 3 where we plot the 2D contours in the  $\alpha - A_s$  plane for  $1/m \leq 0.1$  Mpc. The range of  $A_s$  shown here corresponds to our uniform prior on  $A_s$  motivated in section III B. Clearly this choice of prior is a key for weak-lensing measurement and tightening it more would also strengthen our constraints on  $\alpha$  but we choose a fairly conservative prior in order to secure the consistency with the latest CMB and  $Ly_\alpha$  observations.

The right panel of Fig. 2 shows the same constraints using the SDSS LRGs. Clearly, they are much tighter. The reason for this difference can be understood in the following way. Since we marginalize over the amplitude of the power spectrum within our  $A_s$  priors (and also on the bias in the case of SDSS), our test of gravity basically relies on constraining the shape of the power spectrum. Because the SDSS data we are using cover about 2400 square degrees with accurate photo- $z$  for all of those (and a median redshift of  $z \simeq 0.1$ ), the power spectrum we are using covers the range  $0.02 h/\text{Mpc} \leq k \leq 0.3 h/\text{Mpc}$ . On the other hand, CFHTLS-Wide data, makes uses of only 57 square degrees so that the errors on the same mode are at least 6 times larger due to cosmic variance only. In addition, since cosmic shear is sensitive to a weighted projection of the matter power spectrum (see Eq. 16), the

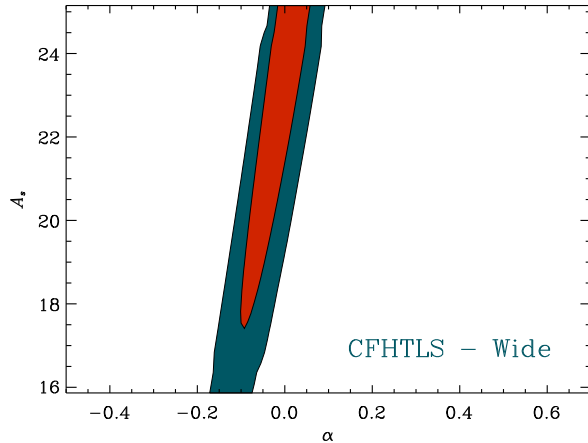


FIG. 3: Likelihood contours at 68% and 95% confidence levels for the normalisation parameter  $A_s$  and  $\alpha$  parameter of the Yukawa type modification to gravity. We consider for this plot only  $1/m \leq 0.1 \text{ Mpc}$  so that we explore the lower part of the left plot in Fig. 2. This plot illustrates the degeneracy between  $\alpha$  and  $A_s$  that is expected given Fig. 1 where we can see that for a given  $m$  and redshift,  $\alpha$  will change the amplitude of  $P(k)$  above a given scale.

shape information is partially erased due to projection effects. This effect could be greatly alleviated if we were able to separate the lenses into various  $z$  planes, *i.e.* to perform tomography, but such a work is still in progress for the CFHTLS survey.

On the other hand, for a given scale  $m$  well probed by both cosmic shear and galaxy clustering data, say  $m = 25 \text{ Mpc}^{-1}$  ( $1/m = 0.04 \text{ Mpc}$ ) CFHTLS performs better than one would expect from simple cosmic variance arguments as is visible in Fig. 4. This figure shows the 1D marginalized likelihood distribution for  $\alpha$  when one sets  $1/m = 0.04 \text{ Mpc}$ . It can be understood as a horizontal slice in Fig. 2 for  $1/m = 0.04 \text{ Mpc}$ . We see in this figure that the 68% uncertainty level for  $\alpha$  using CFHTLS-Wide is only 2.2 bigger than the SDSS ones. This comes from the fact that lensing does not suffer from the bias uncertainty so that within our amplitude prior, lensing is sensitive to the overall amplitude whereas galaxy clustering is not. On the other hand, as illustrated above, if we broaden our normalization prior then the lensing constraints will broaden too.

Fig. 5 displays the 1D 68% and 95% confidence levels on  $r_s$  from the UB model obtained either with the cosmic shear or galaxy clustering observations. Once again, we do not see any evidence for any deviation from standard gravity on the scales we probed. Not surprisingly, we find that as for the Yukawa model, SDSS performs better than CFHTLS in constraining  $r_s$  by about one order of magnitude. Once again, this stems directly from the wider range of scales probed by SDSS.

Our results for the Yukawa model are consistent with

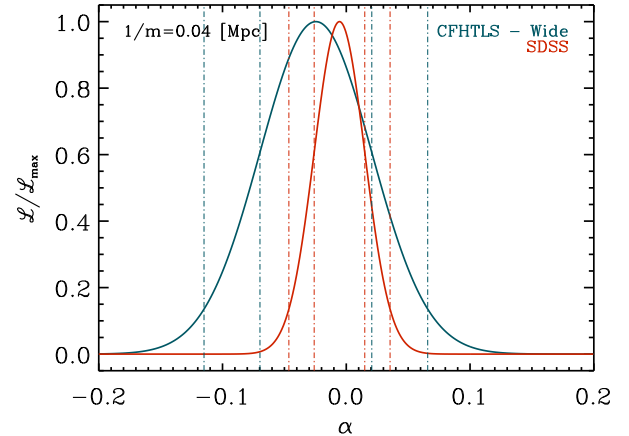


FIG. 4: 1D likelihood distribution for  $\alpha$  when one sets  $1/m = 0.04 \text{ Mpc}$ . Likelihood corresponds to either CFHTLS-Wide or SDSS. Vertical lines correspond to 68% and 95% confidence levels. We can see that for this scale, CFHTLS-Wide constraints are only  $\simeq 2$  times worse than SDSS ones. This can be understood from the fact that lensing is sensitive to the overall normalization of the power spectrum.

those previously published in the literature using the SDSS results [56, 57] or a simpler form of cosmic shear measurements [54]. Our constraints on the UB model are in agreement with those derived using the shear 3-point function [34].

Before discussing further our results, we reassess the theoretical hypothesis underlying our work. First we assume that the background evolution is identical to the  $\Lambda\text{CDM}$  one, which might over-estimate the effects on large scales [55]. Second, we assume that the relation between the deflection angle and the gravitational potential that underlies the convergence power spectrum definition in Eq. 16 is unchanged as compared to GR. Models where this relation does not hold are as such neglected in our approach [16, 35, 43, 46]. Third, we consider deviation from the usual cosmological Poisson equation only and restricted ourselves to some parametric form. Those limitations come from the fact that we choose not to work within the frame of a well defined theory of gravity. As such, our approach may be described as a test of the Poisson equation in a  $\Lambda\text{CDM}$  cosmology.

We did not study explicitly the dependence of our constraints as a function of other parameters. Sealfon et al. [57] showed that their variations within currently observationally allowed bounds had minor effects in this context. Besides, the current constraints are such that most of the major degeneracies of the  $\Lambda\text{CDM}$  are broken. The likelihoods are thus almost Gaussian (see Fig. 10 of Spergel et al. [1]) and marginalizing over a parameter is very close to setting it at its most likely value. In contrast, however, the effects of massive neutrino would



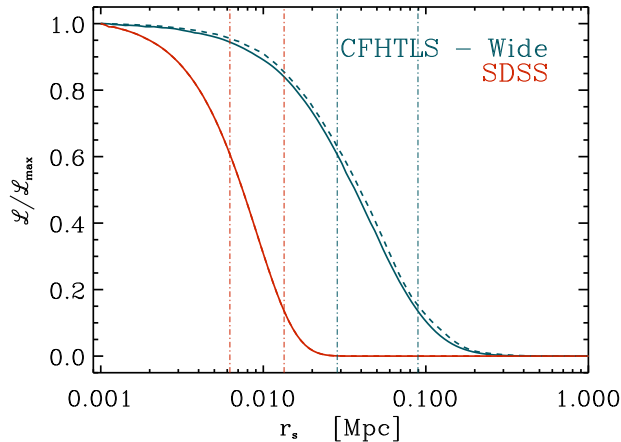


FIG. 5:  $r_s$  likelihood distribution for the UB model. Vertical lines correspond to 68% and 95% confidence levels. Blue lines correspond to CFHLS-Wide constraints whereas red lines correspond to SDSS. The blue dashed line corresponds to the use of the Peacock and Dodds [76] prescription to model non-linearities while the solid line corresponds to the Smith et al. [77] *halofit* prescription.

be degenerated with gravity ones we explore since they also modify the slope of the power spectrum at high  $k$  [44]. On the other hand, as explained before, we also did not quote any constraints in terms of  $\sigma_8$  since this number is obviously very model dependent in our case and a comparison with the standard  $\Lambda$ CDM value would not be meaningful.

The main conclusion of our work is that we did not find any sign of deviation from gravity using large scales probes of the matter perturbations on cosmological scales (between 0.04 and 10 Mpc) at low redshift. Whereas probes of gravity on those scales at low redshifts are scarce, there is another important one we did not use, namely the Integrated Sachs-Wolfe (ISW) effect [80]. However, since our modifications of gravity are totally ad-hoc and do not correspond to any covariant theory, it is unclear yet how meaningful would be our extension to the scales relevant for ISW and we leave that for future work.

Another indirect implication of our work is the fact that the dark matter clustering evolution as probed by cosmic shear measurements, *i.e.* the deflection of lights and the clustering of luminous matter as probed by galaxy surveys provide consistent constraints on theories of modified gravity on 0.04-10 Mpc scales. This agreement has some more quantitative applications that we leave for future work. Note also that within our models, our constraints imply that galactic scale physics is unchanged. Whereas the Yukawa model we discussed was originally introduced as an alternative to dark matter on galactic scales, the extrapolation of our constraint sug-

gests that galactic physics is still ruled by GR.

It is worth noticing that in the context of the linearized theory of gravity, the addition of a Yukawa term corresponds to setting a non-zero mass to the graviton [33, 61, 81]. Our current results do not allow us to place constraint on  $1/m$  if we leave  $\alpha$  vary freely. If we were to set  $\alpha = 1$  as in Choudhury et al. [33], then we obtain the following constraints on the mass of the graviton  $1/m \geq 5.71 \text{ Mpc}$  (95% CL) using CFHTLS, and  $1/m \geq 67.6 \text{ Mpc}$  using SDSS; that is  $m \leq 1.11 \times 10^{-30} \text{ eV}$  using CFHTLS and  $m \leq 9.40 \times 10^{-32} \text{ eV}$  using SDSS.

Further improvements on tests of gravity on cosmological scales should explore more physical assumptions as well as estimators, like tomography. SDSS data show the sky coverage is of primary importance, while CFHTLS-Wide data reveal this survey still covers too narrow a part of the sky and should go one order of magnitude in terms of angular scales. This will be possible soon, with the advent of next generation large scale cosmic shear surveys, such as *e.g.* DES, DUNE, LSST or SNAP [85].

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